Analysis of Polynomial Base Influence in EFG Interpolating Moving Least Square Method Applied to 2D Electromagnetic Scattering

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This work presents a study about behavior of the response of Element Free Galerkin method due to Interpolating Moving Least Square using 2D linear, quadratic and cubic polynomial bases. For this purpose, the 2D electromagnetic scattering problem generated by an infinite dielectric cylinder illuminated by a TM$_z$ plane wave is used. A parametric analysis, comparing numerical with analytical results, is carried out in order to identify the best range of value parameters for each base.

***Index Terms*—Electromagnetic scattering, Element Free Galerkin Method, Meshless Method, polynomial bases.**

I. Introduction

SEVERAL electromagnetic problems are modelled by partial differential equations (PDE), which, in many cases, require a numerical solution. The most popular numerical method for solution of PDE is the Finite Element Method (FEM). However, a frequent problem is the difficult and time-consuming to find an appropriate mesh generation to represent complex geometries with many deformations, discontinuities and/or moving boundaries. An alternative way to solve this type of problem is to use a Meshless Method (MM) [1]. A MM doesn't use a mesh structure and approach the solution using nodes spread throughout the region of interest.

One of the most known and used MM is the Element Free Galerkin (EFG) [2]. Recent works about EFG uses the Interpolating Moving Least Square Method (IMLS) to obtain shape functions that satisfy the Kronecker Delta property [3]-[5]. In the study of electromagnetic scattering, for example, EFG-IMLS allows to impose Absorbing Boundary Condition (ABC) directly into the discrete system. Although an investigation of EFG-IMLS with parametric analysis to a linear base has already been done [6], an EFG-IMLS study using other kind of bases has not been carried out yet.

The present work shows numerical results obtained by EFG-IMLS for a 2D electromagnetic scattering analysis with quadratic and cubic polynomial bases. A comparative study with results from linear polynomial basis is presented.

II. Problem Formulation

The problem on study, illustrated in Fig. 1, consist of scattering analysis of a TM$_z$ plane wave by infinite dielectric cylinder, where Ω_1 is the cylinder domain and Ω_2 is the free space domain, ε_r is the relative electric permittivity of cylinder, Γ_{ABC} represents the global boundary where ABC should be imposed, and E^i is the incident electric field. The 2D study domain is $\Omega = \Omega_1 \cup \Omega_2$.The weak form of the problem, using a first order Bayliss-Turkel ABC, is [6]:

$$\int_{\Omega}\left\{\nabla w\cdot\left(\mu_r^{-1}\nabla E\right)-\left[k_0^2\varepsilon_r wE\right]\right\}d\Omega+\int_{\Gamma_{ABC}}\gamma wE\, d\Gamma_{ABC}=\int_{\Gamma_{ABC}} qw\, d\Gamma_{ABC}, \qquad (1)$$

where μ_r is the relative permeability, E the total electric field, k_0 the wave number, w the weight function (chosen considering the Galerkin method), $\gamma = (1/\mu_r)[1/(2R_b) + jk_0]$ and $q = \gamma E^i + (1/\mu_r)(\nabla E^i)\cdot \boldsymbol{n}$, where $\boldsymbol{n}$ is an unit vector.

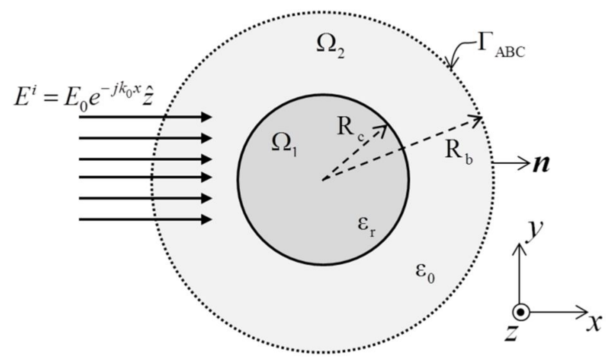

Fig. 1. The problem domain.

In the MM approach, each node, I, is a point $\mathbf{x}_I = (x, y) \in \Omega$ for which a shape function, Φ_I, is associated. A local trial approximation is valid for unknown function E [6]:

$$E^h(\mathbf{x},\mathbf{x}_I)=\sum_{i=1}^{m} p_i(\mathbf{x}_I)a_i(\mathbf{x})\equiv \mathbf{p}^T(\mathbf{x}_I)\,\mathbf{a}(\mathbf{x}), \qquad (2)$$

where m represents the number of monomial terms in the polynomial base $\mathbf{p}^T(\mathbf{x})$ and $\mathbf{a}(\mathbf{x})$ are the unknown polynomial coefficients. $\mathbf{p}^T(\mathbf{x}) = [1\ x\ y]$ for a 2D linear basis, $\mathbf{p}^T(\mathbf{x}) = [1\ x\ y\ x^2\ xy\ y^2]$ for a 2D quadratic basis and $\mathbf{p}^T(\mathbf{x}) = [1\ x\ y\ x^2\ xy\ y^2\ x^3\ x^2y\ xy^2\ y^3]$ for a 2D cubic basis.

In IMLS approximation presented in this work, coefficients $\mathbf{a}(\mathbf{x})$ are determined by minimizing a weighted discrete L^2 norm, for which weight function is:

$$W(r) = 1/(r^n + \beta^n), \qquad (3)$$

where β is a constant with a very small value to ensure no singularity, n is a constant adjusted to improve the accuracy and $r = \sqrt{[(x-x_I)/\alpha]^2+[(y-y_I)/\alpha]^2}$ is the support radius of circular influence domain of each node.

III. Numerical Results

To evaluate the influence of polynomial basis in the EFG-IMLS numerical results for 2D linear, quadratic and cubic bases, the following constants were considered: vacuum wavelength λ_0 = 3m, $R_c = 0.25\lambda_0$, $R_b = 0.5\lambda_0$, $\mu_r = \mu_0$ and $\varepsilon_r = 2-j$ (lossy dieletric simulated by complex permittivity). The

EFG-IMLS accuracy was compared to Analytical Solution (AS) by the following L² norm error:

$$EL^2 = \sqrt{\iint_\Omega \left| E^{EFG-IMLS} - E^{AS} \right|^2 d\Omega} \Big/ \sqrt{\iint_\Omega \left| E^{AS} \right|^2 d\Omega}. \qquad (4)$$

All analyzes presented in this work consider four integration points per rectangular cell. The first analysis is the variation of Number of Node (NN), whose response is shown in Fig. 2. The parameters used for EFG-IMLS are 6652 Gauss Integration Points (GIP), β = 0.1, n = 8 and α = 3. As can be observed, the EL^2error decreases as NN increases for all bases until a certain value from which there is no significant variation of EL^2. This happens for 762 nodes for linear (EL^2= 3.5%) and cubic bases (EL^2= 3.1%) and 1601 nodes for quadratic basis (EL^2= 6.0%). Using these values of NN and the same n, β, α from previous simulation, a GIP variation analysis is shown in Fig. 3. One sees that the bases analyzed have the same behavior, with EL^2 decrease until approximately GIP = 8000, when the error stabilizes. The minimum EL^2 errors obtained are with 2836 GIP for linear base (EL^2= 3.01%), 7876 for quadratic base (EL^2= 5.19%) and 2836 for cubic base (EL^2= 2.24%). These GIP values are used to verify the EL^2 error with n, β, α variation. Fig. 4 shows that the bases present different n value ranges for which the EL^2 error is small. Linear basis has a range of n = 8 to 23, quadratic basis exhibit a range of n = 8 to 12 and cubic basis, n = 5 to 9. For the β used in simulation, as seen in Fig. 5, linear basis maintain small EL^2 error for all values, quadratic base for β = 0.04 to 0.2, and cubic base for β greater than 0.07. For the α variation, shown in Fig. 6, linear basis have similar EL^2 error for all values. Quadratic basis presents small EL^2 errors for α greater than 1.3 and cubic base for α greater than 2.1.

The results presented in this work suggest the quadratic basis doesn't reach the same precision level of linear and cubic bases. In general, linear basis have a wider range of good values for n, β, α parameters to choice than others bases. This characteristic makes linear base more flexible to obtain accurate results in a numerical analysis. However, for a parametric optimization with the objective to minimize EL^2 error, the cubic basis could be more appropriate due to its narrower range of parametric values. A study to show the impact of this characteristic and compare the performance of optimal parameters obtained by an optimization algorithm for linear and cubic bases does not exist yet.

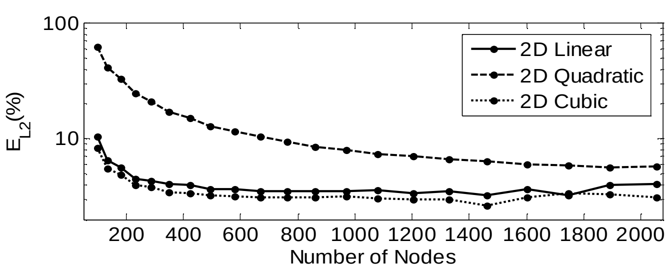

Fig. 2. Norm L² error for variation of number of nodes.

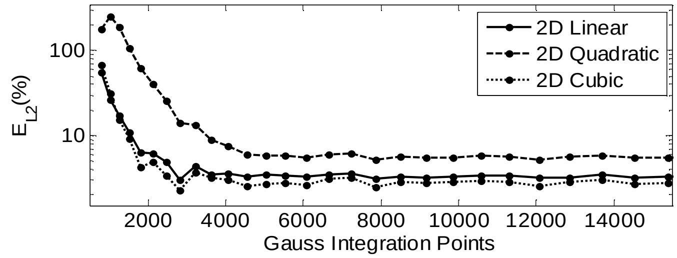

Fig. 3. Norm L² error for variation of GIP.

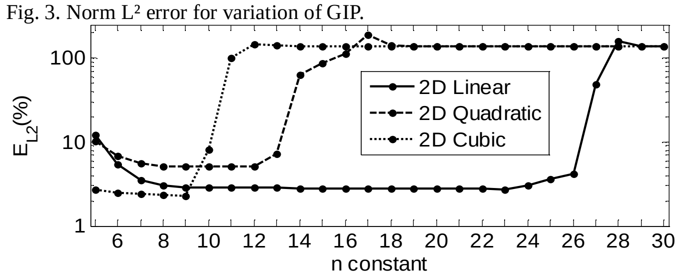

Fig. 4. Norm L² error for variation of *n*.

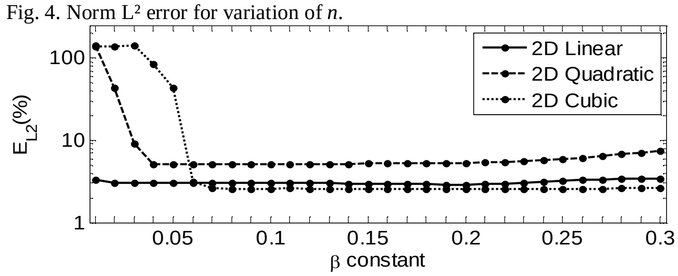

Fig. 5. Norm L² error for variation of β.

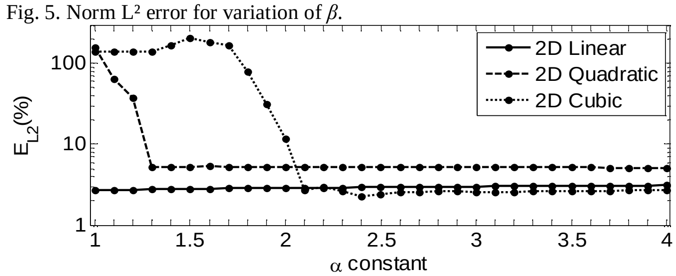

Fig. 6. Norm L² error for variation of α.

ACKNOWLEDGEMENT

This work was partially supported by FAPEMIG, CAPES, CNPq and CEFET-MG.

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